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*A Survey of Alternative Methodologies for Estimating Potential Output and the Output Gap**

In this paper, we survey some techniques proposed in the literature to measure potential output. Given the reported shortcomings of univariate approaches and mechanical filters, we focus on three simple multivariate methodologies: the multivariate Beveridge-Nelson methodology (MBN), Cochrane's methodology (CO), and the structural VAR methodology with long-run restrictions (LRRO). These methodologies are presented and then applied to U.S. data. The results show that the LRRO estimates provide significant evidence that permanent shocks have more complex dynamics than the random walk assumed in CO and MBN approaches. As in other studies, estimates of the output gap remain imprecise.

1. Introduction

Most structural macroeconomic models that are used for forecasting and policy analysis require an estimate of potential output. In these models, the gap between actual and potential output is a key variable determining the evolution of prices and wages. A level of real GDP above (below) potential will often be seen as a source of inflationary (disinflationary) pressures and a signal that the monetary authorities should tighten (ease) monetary conditions.[1] Potential output is often associated with the permanent com-

*The authors would like to thank Alain DeSerres, Pierre Duguay, Paul Fenton, Robert Lafrance, René Lalonde, John Murray, Alain Paquet, Simon van Norden and two anonymous referees for useful comments. They also want to thank Jennifer Page and Robert Vigfusson for their excellent research assistance. The views expressed in this paper do not necessarily represent those of the Bank of Canada. The authors are solely responsible for the paper's content.

[1]For a discussion of how the estimation of potential output can affect the formulation of monetary policy, see Boschen and Mills (1990) or Laxton and Tetlow (1992).

Journal of Macroeconomics, Summer 1999, Vol. 21, No. 3, pp. 577–595

0164-0704/99/$1.50

ponent (or trend) of output. The output gap then corresponds to the transitory component of output.

Since the publication of Nelson and Plosser's (1982) influential paper suggesting that output series are best characterized as integrated series, there has been increasing recognition that measuring the permanent component of output, or potential output, with any degree of accuracy is a difficult task. The presence of a stochastic permanent component implies that potential output cannot be treated as a deterministic trend. As a result, various methods have been proposed to uncover the permanent and transitory components of output.

One of these methods consists of using mechanical filters such as the Hodrick-Prescott (HP) filter or the band-pass filter (BK) proposed by Baxter and King (1995). However, mechanical filters have been criticized. For example, Harvey and Jaeger (1993) and Cogley and Nason (1995) show that spurious cyclicality can be induced by the HP filter when it is used with integrated or nearly integrated data. Guay and St-Amant (1996) reach the more general conclusion that the HP and BK filters perform poorly in identifying the cyclical component of time series that have a spectrum or pseudo-spectrum with Granger's typical shape, that is, the shape characteristic of most macroeconomic time series.

Baxter and King (1995) and others note that two-sided filters such as the HP and BK filters become ill-defined at the beginning and the end of samples. For this reason, they recommend discarding three years of quarterly data at both ends of the sample when using the HP filter. Van Norden (1995) stresses the fact that this is a very significant limitation for policy-makers interested in estimating the current level of the output gap.

Another strategy for identifying the permanent and transitory components of output involves the use of univariate statistical techniques such as the unobserved components approach suggested by Watson (1986) and the Beveridge-Nelson (1981) method. However, Quah (1992) has shown that "without additional ad hoc restrictions those [univariate] characterizations are completely uninformative for the relative importance of the underlying permanent and transitory components."

Partly in response to this criticism, a variety of multivariate methods have been proposed to generate permanent-transitory decompositions that appear less arbitrary, or that at least can be given a structural interpretation. One example is a decomposition method proposed by Cochrane (1994) which is based on the permanent-income theory and uses consumption to define the permanent component of output. Multivariate extensions of the Beveridge-Nelson decomposition method (MBN) have also been applied to identify the trend component of output (Evans and Reichlin 1994). However, a major restriction in the univariate context, which is maintained in the multivariate extensions, is that the permanent component of output behaves like a random walk. This assumption is difficult to reconcile with the widely held view that the permanent component of output is, at least in part, driven by technological innovations. As stressed by King et al. (1991), "productivity shocks set off transitional dynamics, as capital is accumulated and the economy moves towards a new steady-state." Lippi and Reichlin (1994) go even further, arguing that modeling the trend in output as a random walk is inconsistent with standard views concerning the dynamics of productivity shocks. Adjustment costs on capital and labor, learning and diffusion processes, habit formation and time to build all imply richer dynamics than a random-walk process for technology shocks.

The structural vector autoregression methodology based on long-run restrictions imposed on output (LRRO) proposed by Blanchard and Quah (1989), Shapiro and Watson (1988), and King et al. (1991) does not constrain the short-run dynamics of the permanent component of output. Instead, it allows for a permanent component incorporating an estimated diffusion process for permanent shocks that can differ from the random walk. It will often be interesting for researchers and policymakers to include the dynamics of permanent shocks in potential output (as opposed to the output gap) since they are more likely to reflect the production capacity of the economy.

In this paper, we compare the Cochrane (CO), MBN, LRRO approaches using U.S. data and ask whether the choice of a particular methodology really makes a difference statistically. In particular, since the differences among these methods come from the presence of short-run dynamics in the permanent component, we try to determine whether this short-run dynamics is statistically significant. We find that the answer is "yes" in terms of impulse responses or when the entire output gap series is considered, but "not really" when one is interested in comparing output gaps at specific points in time. In the latter case, the estimation of potential output and of the output gap is indeed imprecise.[2] Another interesting result is that, of the methods we consider, only the LRRO generates an output gap with a peak at business-cycle frequencies as defined by Burns and Mitchell (1946).

What are the implications of these results for researchers and policymakers? If one is interested in correlations with other variables or in computing the dynamic properties of potential output (trend output) and the output gap (structural cyclical component), the choice of methodology matters. On the other hand, if one is interested in assessing the output gap and potential output at a specific point in time, these methodologies are not likely to provide answers that are statistically different. Finally, the LRRO ap-

[2]This is consistent with recent results reported by Staiger, Stock and Watson (1996) for the estimation of the NAIRU.

proach should be preferred if one believes that the dynamics of permanent shocks should be included in potential output.

In a recent paper, Cogley (1995) compares non-structural approaches (CO, MBN, HP, and BK filters) to measure the trend and the cycle of simulated output processes generated with an RBC model. He reaches the conclusion that the CO approach dominates the others. While Cogley is interested in the performance of non-structural approaches to measure structural cycles, we are interested in comparing a structural approach (LRRO) with non-structural approaches (CO and MBN) to decompose the observed U.S. real output into trend and cyclical components.

The rest of the paper is organized as follows. Section 2 provides a theoretical comparison of the CO, MBN and LRRO approaches. Section 3 presents an empirical comparison of these approaches. In particular, we assess the robustness of the results by considering a number of alternative specifications of the VAR representation. We conclude in Section 4.

2. Methodologies Used for Estimating the Trend in Output

The Approach Based on the LRRO Methodology

In this section, we briefly present the LRRO decomposition methodology involving long-run identifying restrictions (LRRO) and explain how it can be used to estimate potential output.[3]

Let Z_t be a $n \times 1$ stationary vector including a n_1-vector of $I(1)$ variables and a n_2-vector of $I(0)$ variables such that $Z_t = (\Delta X'_{1t}, X'_{2t})'$.[4] By the Wold decomposition theorem, Z_t can be expressed as the following reduced form:

$$Z_t = \delta(t) + C(L)\varepsilon_t , \tag{1}$$

where $\delta(t)$ is deterministic, $C(L) = \Sigma_{i=0}^{\infty} C_i L^i$ is a matrix of polynomial lags, $C_0 = I_n$ is the identity matrix, the vector ε_t is the one-step-ahead forecast errors in Z_t given information on lagged values of Z_t, $E(\varepsilon_t) = 0$, and $E(\varepsilon_t \varepsilon'_t) = \Omega$ with Ω positive definite. We suppose that the determinantal polynomial $|C(L)|$ has all its roots on or outside the unit circle, which rules out non-fundamental representations emphasized by Lippi and Reichlin (1993).

Equation (1) can be decomposed into a long-run component and a transitory component:

[3]See Watson (1993) for a more detailed presentation of the LRRO approach.

[4]$I(d)$ denotes a variable that is integrated of order d.

$$Z_t = \delta(t) + C(1)\varepsilon_t + C^*(L)\varepsilon_t , \tag{2}$$

where $C(1) = \Sigma_{i=0}^{\infty} C_i$ and $C^*(L) = C(L) - C(1)$. This decomposition corresponds to the multivariate Beveridge-Nelson decomposition (see Evans and Reichlin 1994 and King et al. 1991). We define $C_1(1)$ as the long-run multiplier of the vector X_{1t}. If the rank of $C_1(1)$ is less than n_1, there exists at least one linear combination of the elements in X_{1t} that is $I(0)$. In other words, there exists at least one cointegration relationship between these variables (see Engle and Granger 1987).

The LRRO approach assumes that Z_t has the following structural representation:

$$Z_t = \delta(t) + \Gamma(L)\eta_t , \tag{3}$$

where η_t is a n-vector of structural shocks, $E(\eta_t) = 0$, and $E(\eta_t\eta_t') = I_n$ (a simple normalization). From the estimated reduced form, we can retrieve the structural form (3) using the following relationships: $\Gamma_0\Gamma_0' = \Omega$, $\varepsilon_t = \Gamma_0\eta_t$, and $C(L) = \Gamma(L)\Gamma_0^{-1}$.

The long-run covariance matrix of the reduced form is equal to $C(1)\Omega C(1)'$. From (2) and (3) we have

$$C(1)\Omega C(1)' = \Gamma(1)\Gamma(1)' . \tag{4}$$

This relation suggests that we can identify matrix Γ_0 with an appropriate number of restrictions on the long-run covariance matrix of the structural form. Blanchard and Quah (1989) and Shapiro and Watson (1988) use long-run restrictions to identify shocks with $C(1)$ having full rank. King et al. (1991) work in a context where the rank of $C(1)$ is less than n_1, and they use cointegration restrictions.

Let us assume that the log of output is the first variable in the vector Z_t. It is then equal to:

$$\Delta y_t = \mu_y + \Gamma_y^p(L)\eta_t^p + \Gamma_y^c(L)\eta_t^c , \tag{5}$$

where η_t^p is the vector of permanent shocks affecting output and η_t^c is the vector containing shocks having only a transitory effect on output. Potential output based on the LRRO method is then

$$\Delta y_t^p = \mu_y + \Gamma_y^p(L)\eta_t^p . \tag{6}$$

Thus, "potential output" corresponds to the permanent component of output. The part of output due to transitory shocks is defined as the "output gap."

Comparison with Other Multivariate Methods

In this section, we examine the features of two alternatives to the LRRO approach: the multivariate Beveridge-Nelson decomposition (MBN) and Cochrane's decomposition (CO).[5]

The MBN decomposition defines potential output as the level of output that is reached after all transitory dynamics have worked themselves out. With reference to Equation (2), where output is the first element of Z_t, we write the following decomposition:

$$\Delta y_t = \mu_y + C_y(1)\varepsilon_t + C^*_y(L)\varepsilon_t . \tag{7}$$

Potential output is defined by the first two terms on the right-hand side of (7):

$$\Delta y_t^p = \mu_y + C_y(1)\varepsilon_t . \tag{8}$$

Potential output is thus simply a random walk with drift.

Cochrane (1994) uses a two-variable VAR including GNP and consumption to identify the permanent and transitory components of output. The bivariate representation is augmented with lags of the ratio consumption to GNP. The permanent-income theory implies that consumption is a random walk (for a constant real interest rate). In addition, if we assume that GNP and consumption are cointegrated, then fluctuations in GNP with consumption unchanged must be perceived to be transitory. It is on this basis that Cochrane decomposes GNP into permanent and transitory components. To extract potential output, the errors of the VAR are orthogonalized so that consumption does not respond contemporaneously to GNP shocks.

Cochrane shows that if GNP and consumption are cointegrated and consumption is a random walk, identification based on the LRRO method and conventional orthogonalization (i.e., a Choleski decomposition) amounts to the same thing. Moreover, if consumption is a pure random walk, Cochrane's decomposition corresponds exactly to the Beveridge-Nelson decomposition. The moving-average representation of these processes when the log of consumption is a random walk can be written as

$$\Delta y_t = \mu_y + C_y(1)\varepsilon_t + C^*_y(L)\varepsilon_t ; \tag{9}$$

$$\Delta c_t = \mu_c + C_c(1)\varepsilon_t ; \tag{10}$$

[5]See Cogley (1995) for another comparison of the MBN and CO methodologies.

where $\mu_y = \mu_c$ and $C_y(1) = C_c(1)$. Potential output is defined as the first two terms on the right-hand side of the output equation, these terms being equal to the change in consumption. A multivariate Beveridge-Nelson decomposition including GNP and consumption would have the same form.

In order to compare the LRRO approach based on long-run restrictions with the CO and MBN approaches, let us first rewrite structural form (3) in terms of the log of GNP (y_t) and the log of consumption (c_t) decomposed between permanent and transitory shocks assuming that y_t and c_t are cointegrated:

$$\Delta y_t = \mu_y + \Gamma_y^p(1)\eta_t^p + \Gamma_y^{p*}(L)\eta_t^p + \Gamma_y^c(L)\eta_t^c , \qquad (11)$$

$$\Delta c_t = \mu_c + \Gamma_c^p(1)\eta_t^p + \Gamma_c^{p*}(L)\eta_t^p + \Gamma_c^c(L)\eta_t^c ; \qquad (12)$$

where $\Gamma^p(1)$ is the long-run multiplier of permanent shocks and $\Gamma^{p*}(L) = \Gamma^p(L) - \Gamma^p(1)$ is their transitory component. The MBN method considers only the first component of the permanent shocks plus the drift term, that is, $\mu + \Gamma_y^p(1)\eta_t^p$. In this case, potential output is restricted to be a random walk. The LRRO approach is different in that it also includes the dynamics of permanent shocks ($\Gamma^{p*}(L)$).

With the CO approach, potential output is constrained to be a random walk to the extent that consumption is a random walk. Indeed, the validity of the permanent-income hypothesis would imply that the last two terms of the consumption equation are equal to zero and that $\Gamma_y^p(1) = \Gamma_c^p(1)$. It is not clear to what the CO decomposition corresponds if consumption is not a random walk.[6] Cochrane (1994) notes that the measure of potential output obtained on the basis of the CO method would be equivalent to the one obtained from the LRRO approach if the transitory effect of permanent shocks to GNP and consumption were exactly the same, that is, if $\Gamma_y^{p*}(L) = \Gamma_c^{p*}(L)$ and $\Gamma_y^p(1) = \Gamma_c^p(1)$. However, these restrictive conditions are unlikely to occur in practice.

As pointed out by Lippi and Reichlin (1994), modeling the trend in output as a random walk is inconsistent with most economists' interpretation of productivity growth. Indeed, it is generally believed that technology shocks are absorbed gradually by the economy. Adjustment costs for capital and labor, learning and diffusion processes, habit formation, and time to

[6]Stochastic growth models—such as in King, Plosser and Rebelo (1988) or King et al. (1991)—imply that the ratio of the log of GNP to the log of consumption is stationary but that consumption is not a random walk because the real interest rate is not constant. In these models, the transitory component of permanent shocks to consumption is not equal to zero. The LRRO decomposition is compatible with the prediction of these models.

build all imply richer dynamics than a random walk for these shocks. Working in a univariate framework, Lippi and Reichlin must constrain the dynamic of the trend to follow a particular shape (S-shape dynamic) in order to identify the trend and cyclical components. Again, a decisive advantage of the LRRO approach is that it lets the data determine the shape of the diffusion process of permanent shocks.[7]

One implication of defining potential output as a random walk with drift is that when the contemporary effect of a positive permanent shock is smaller (greater) than its long-run effect, the output gap, defined as observed output minus potential, is negative (positive). For example, a positive technological shock whose short-term impact is smaller than its long-term impact will cause a transitory negative output gap. Many researchers and/or policymakers will find that this feature of the MBN and CO approaches (in the later case under the assumption that consumption is a random walk) reduces their attractiveness. It will often appear preferable to include the diffusion process associated with permanent shocks in potential output since the economy is likely to remain on its production possibility frontier as adjustments unfold. In many models used for policy analysis there will then be no reason for trend inflation to change.

3. Empirical Results

In this section, we first present applications of the LRRO approach to U.S. data. Results are discussed and compared with those obtained from applications of the MBN and the CO approaches. We then compare the spectra of the output gaps estimated on the basis of these methodologies.

Applications to U.S. Data

For our applications of the LRRO methodology we assume that output in first differences (Δy) follows a stationary stochastic process responding to two types of structural shocks: permanent (η^P) and transitory (η^T). The difference between the levels of output (y) and real consumption (c) is included in the estimated VAR (VAR2) to help capture the cyclical component of output. The use of this variable, which we suppose stationary, facilitates the comparison between the different methodologies.[8] We also consider a case

[7]Kuttner (1994) proposes a method based on the univariate unobserved stochastic trend decomposition of Watson (1986) augmented with a Phillips-curve equation. As with the Beveridge-Nelson decomposition, Kuttner's approach constrains potential output to follow a random-walk process.

[8]Preliminary estimates suggest that the difference between real GDP and real consumption of non-durables and services may not be stationary in other countries (e.g., Canada, Germany, and the United Kingdom). It may thus be difficult to use that series in these cases in the same way it is used in the present paper.

where we add a nominal variable to the information set (VAR3) as recommended by King et al. (1991). In a vector form, the structural shocks and the variables used in these VARs can be expressed as

$$\eta_t = \begin{bmatrix} \eta_t^P \\ \eta_t^T \end{bmatrix}$$

and

$$Z_t = \begin{bmatrix} \Delta y_t \\ (y - c)_t \end{bmatrix}$$

or

$$\eta_t = \begin{bmatrix} \eta_t^P \\ \eta_t^{T1} \\ \eta_t^{T2} \end{bmatrix}$$

and

$$Z_t = \begin{bmatrix} \Delta y_t \\ (y - c)_t \\ \Delta i_i \end{bmatrix} .$$

We use quarterly data on the U.S. real GDP and real consumption of non-durables and services (the series used by Cochrane) and the federal funds rate (which we assume to be non-stationary (it is in first differences in the VAR).[9] Our sample extends from the first quarter of 1963 to the second quarter of 1997.

The autoregressive reduced-form VAR of the model is first estimated:

$$Z_t = \sum_{i=1}^{q} \Pi_i Z_{t-i} + \varepsilon_t ,$$

with q the number of lags and ε_t a vector of estimated residuals with $E(\varepsilon_t \varepsilon_t') = \Omega$.

It is crucial that the estimated VARs include a sufficient number of lags. Indeed, Monte Carlo simulations by DeSerres and Guay (1995) show that using a lag structure that is too parsimonious can significantly bias the

[9]All series were taken from the data base of Data Resources Incorporated (DRI).

estimation of the structural components. These authors also find that information-based criteria, such as the Akaike and Schwarz criteria, tend to select an insufficient number of lags, while Wald or likelihood-ratio (LR) tests, using a general-to-specific approach, perform better. Accordingly, we applied such an LR test and obtained results going from four to eight lags for the different VARs.

The LRRO approach involves the identification of structural shocks (η_t) from reduced-form shocks (ε_t) and their variance. For this, we need to provide enough identifying restrictions to evaluate the elements in Γ_0. In the two-variable system, Γ_0 has four elements. Given that Ω is symmetric, we need to impose one additional restriction. The matrix of long-run effects of reduced-form shocks, $C(1)$, is related to the equivalent matrix of structural shocks, $\Gamma(1)$, as follows:

$$\Gamma(1) = C(1)\Gamma_0 ,$$

where the matrix $C(1)$ is calculated from the estimated VAR.[10] The additional identifying restriction, which is imposed on $\Gamma(1)$, is simply that shocks affecting potential output are the ones having a permanent impact on real GDP. In the cases of higher order VARs, we impose that $\Gamma(1)$ is triangular to identify the system. The transitory component then corresponds to the sum of each component originating from transitory shocks.[11]

To verify that our results are not significantly affected by the problem of shock aggregation stressed by Braun and Mittnik (1993) and Faust and Leeper (1997), we consider additional VAR specifications. VAR4 is estimated with the first difference of the unemployment rate in addition to the three variables included in VAR3. This series was obtained from DRI. VAR5 adds a measure of the nominal effective exchange rate vis-à-vis the U.S. six most important trading partners (we use trade weighted indexes). We present results for VAR4 and VAR5 when appropriate. Essentially, these later VARs give results very similar to VAR3, which suggest that what we obtain with VAR3 does not suffer from the shock aggregation problem.[12]

[10]Faust and Leeper (1997) argue that assumptions have to be made to obtain valid inferences with the method based on long-run restrictions. One assumption that can be made and that renders inference valid (according to Faust and Leeper) is that the reduced-form impulse responses are zero passed a certain horizon. We verified that the estimation of $C(1)$ was not sensitive to the choice of the truncation point for the estimation of the moving-average representation, which suggests that this assumption is not too constraining.

[11]Of course, other identification restrictions could have been considered. However, in this paper we are interested in decomposing output into transitory and permanent components which corresponds to the set of assumptions that we used.

[12]We considered even larger VARs including variables such as money and the price of oil and obtained essentially the same results as with VAR3, VAR4, and VAR5.

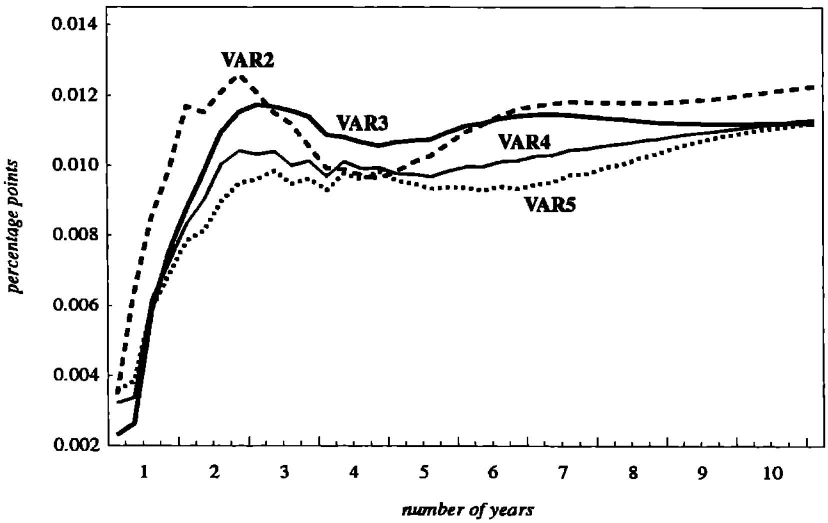

Figure 1.
Responses of Real GDP to the Permanent Shocks

Figure 1 presents the impulse responses of output to a permanent shock of one standard deviation in size in all four cases. (Responses to transitory shocks are presented in Appendix 1). The horizontal axis represents the number of years. The important message of Figure 1 is that permanent shocks are characterized by statistically significant diffusion processes; that is, permanent shocks have a richer dynamic than the random walk.[13] As mentioned above, this could be due to factors such as adjustment costs on capital and labor, learning, habit formation, or time to build. Note that nothing constrains our estimation procedure to find such a diffusion process; that is, this exercise can be seen as a test of the assumption that potential output follows a random-walk process. One implication of the rejection of that assumption is that methods that do not take into account the diffusion process of permanent shocks could miss an important part of potential output.

As mentioned in Section 2, models in which the permanent component of output is a random walk imply that the economy is below (above) potential in the transition period following a permanent positive (negative) shock to output. To the extent that the transition primarily reflects factors associated

[13]Confidence intervals, generated with Monte-Carlo simulations in RATS involving 1000 draws, confirm that the diffusion process is statistically significant at conventional levels in all four cases.

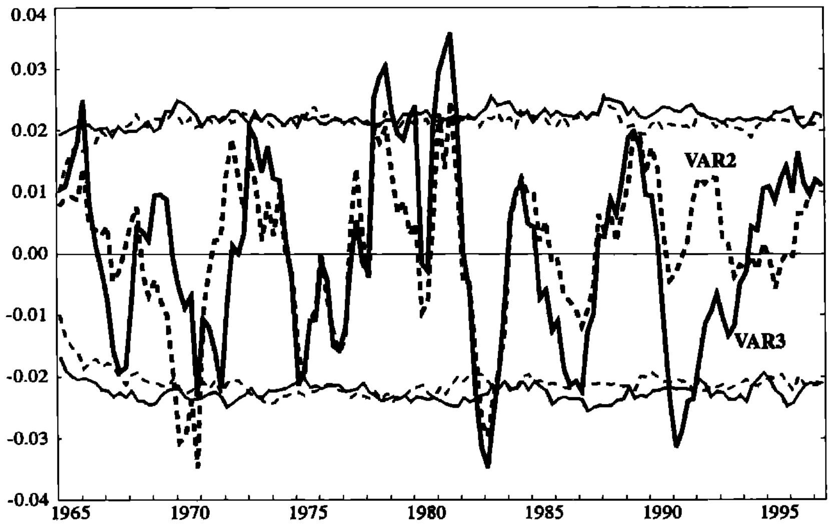

Figure 2.
Uncertainty Surrounding the Estimation of the Output Gap

with an adjustment in the supply side of the economy, assuming that potential output follows a random walk can be misleading. It could, in particular, provide misleading signals about the extent of inflationary pressures in the economy.

Figure 2 shows the output gaps calculated on the basis of the LRRO methodology in the bivariate and trivariate cases. The 90% confidence intervals are the solid band for VAR3 and the dotted band for VAR2. The reported output gaps are clearly different. Indeed, the correlation between these series is only 0.65 (see Table 1). This suggests that interest rates are informative with respect to the transitory component of output and that limiting the system to two variables induces the type of shock aggregation problem stressed by Faust and Leeper (1997). It is interesting to note that, while the bivariate model implies that the early 1990s recession was driven mainly by permanent shocks, the trivariate application suggests that there was a large output gap at that time.

Figure 3 shows the output gaps calculated on the basis of the LRRO, the MBN and the CO approaches (the NBER recession periods are also presented as a reference item). In the cases of the LRRO and MBN approaches, we present trivariate applications. The LRRO and MBN applications are based on the same estimated VARs so that only different identifying assumptions distinguish the two. For the MBN application, we simply as-

TABLE 1. *Correlations between the Output Gaps (Standard errors are shown in parenthesis)*

	LRRO (2 variables)	LRRO (3 variables)	CO	MBN (2 variables)	MBN (3 variables)
LRRO (2 variables)	1	—	—	—	—
LRRO (3 variables)	0.65 (0.09)	1	—	—	—
CO	0.43 (0.13)	0.50 (0.13)	1	—	—
MBN (2 variables)	0.62 (0.09)	0.39 (0.14)	0.03 (0.19)	1	—
MBN (3 variables)	0.46 (0.11)	0.75 (0.07)	0.32 (0.17)	0.57 (0.09)	1

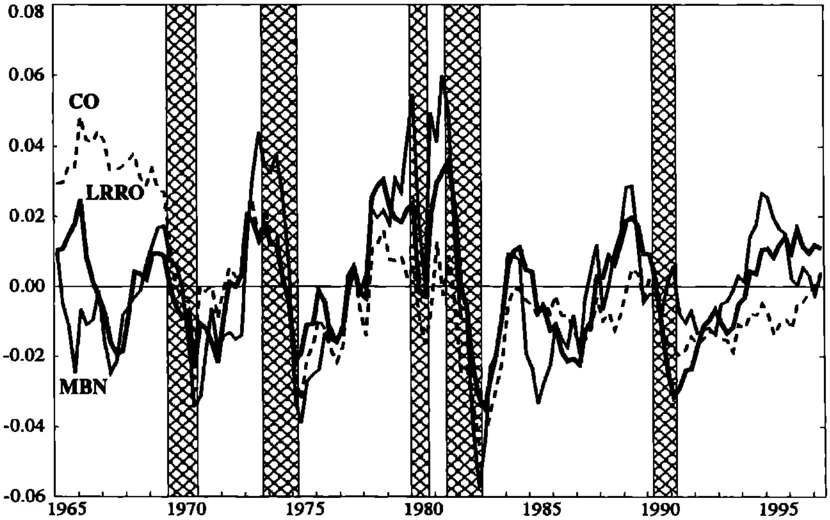

Figure 3.
Output Gaps Calculated on the Basis of the LRRO, CO, and MBN Approaches

sume that potential output corresponds to the accumulation of the long-run effect of reduced form shocks plus the drift in output. The application of the CO approach assumes that the output gap corresponds to the difference between the log of real GDP and the log of real consumption of non-durables and services. This supposes that the permanent-income hypothesis is a good approximation of the real DGP for consumption. As mentioned in Section 2, if the permanent-income hypothesis were not true, it is not clear what the CO decomposition would correspond to.

Table 1 shows that the cross-correlations between the output gaps calculated on the basis of the different approaches are relatively small. The fact that the LRRO approach allows for a gradual diffusion of permanent shocks into potential output accounts for part of this. Also, the correlation between the output gaps identified on the basis of the CO and MBN methodologies is rather small, indicating that consumption may not be a random walk. This is consistent with results reported in Watson (1993), showing that the spectrum of the first difference of consumption has a peak at business-cycle frequencies. In Section 2, we noted that if consumption followed a random-walk process, the CO and MBN methodologies would give identical results.[14]

Although the correlations between the different output gaps appear to be relatively small, it might still be difficult to discriminate among the different measures of the output gap for specific points in time. The monetary authority is, of course, interested in whether it is possible to discriminate among different methodologies for the estimation of the current output gap and in knowing how precise this estimation is. Figure 2 shows that there is indeed a substantial amount of uncertainty surrounding the estimation of the output gap.[15] Staiger, Stock and Watson (1996), using a different methodology, reach a similar conclusion concerning the estimation of the NAIRU. This uncertainty should probably be taken into account by policymakers who use the output gap to guide their decisions. It may be possible to reduce the uncertainty by taking into account other indicators of inflationary pressures, such as money growth and measures of wage pressures. Figures 2 and 3 also suggest that it is difficult to distinguish statistically among the different methodologies when estimating the output gap for specific points in time.

[14]The correlations between gaps calculated on the basis of VAR3 to VAR5 are high, ranging from 81% to 96%.

[15]We performed the LRRO methodology with VARs constrained on the basis of the methodology suggested by Lütkepohl and Poskitt (1996). This allowed for the elimination of about 40% of the parameters in the VARs. The resulting output gaps were very similar to the ones estimated with unconstrained VARs, but the size of their confidence bands was only slightly smaller.

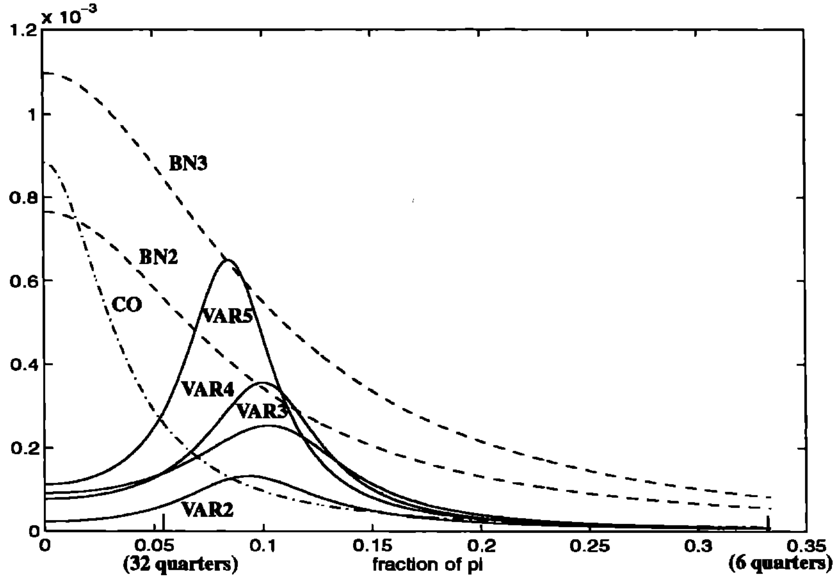

Figure 4.
Spectra of the Output Gaps

Spectra Analysis

Loosely speaking, the spectrum of a series is that series expressed as the integral of random periodic components that are mutually orthogonal. The total area below the spectrum corresponds to the variance of the series. The height of the spectrum at a given frequency shows to what extent cycles of a certain length contribute to the variance of a series. Consequently, the spectrum is an interesting tool for studying the properties of a time series in the frequency domain. We use a parametric estimator of the spectra for which ARMA processes were fitted.[16] Figure 4 shows the estimated spectra of the CO, MBN (bivariate and trivariate cases) and LRRO (VAR2 to VAR5).

All the spectra resulting from the LRRO output gaps have their peak at business-cycle frequencies as defined by Burns and Mitchell (1946), that is, frequencies corresponding to cycles lasting between 6 and 32 quarters. This result appears robust to the number of variables included in the VAR. Note, however, that we verified that reducing the sample size would make

[16]For an introduction to spectral analysis see Hamilton (1994). The order of these processes was determined on the basis of the Akaike criteria.

it less robust, which might simply reflect the difficulty to estimate the spectra with precision on the basis of a smaller sample. The spectra of the CO gap and those of the MBN gaps have their peak at frequency zero. Indeed, these spectra have the typical Granger shape—see Granger (1966). The CO gap, in particular, appears to be characterized by very long cycles.[17]

Of course, the fact that only the LRRO approach appears to generate spectra peaking at business-cycle frequencies does not necessarily imply, in itself, that it is a better method to measure the business cycle. Nevertheless, we feel that many practitioners would prefer to use a method that does not include a very high proportion of very long cycles in the measured output gap. In particular, many will find attractive a statistically-based method giving an output gap compatible with the NBER definition of what business-cycles are, that is, cycles lasting from 6 to 32 quarters. The LRRO gaps have most of their power at those frequencies. Also, very long cycles such as those prominent in the other measures will usually be seen by policymakers such as central bankers as having little to do with their macroeconomic policy instruments. Indeed, most central bankers would think that monetary policy has to do with business-cycles such as those compatible with the NBER definition.

4. Conclusions

In this paper, we compared different techniques that are used to measure potential output. We started with a brief explanation of why we think that mechanical filters such as the Hodrick-Prescott filter and the band-pass filter proposed by Baxter and King (1995) perform poorly in accomplishing this task. We then compared the LRRO approach based on long-run restrictions with two alternative multivariate approaches: the one proposed by Cochrane (1994) and the multivariate Beveridge-Nelson methodology. We argued that one advantage of the approach based on long-run restrictions is that it allows for estimated transitional dynamics following permanent shocks.

The applications considered in this paper provide evidence that there is a statistically significant gradual diffusion process associated with permanent shocks and that the output gap series estimated on the basis of the

[17]It is well known that including more variables in a VAR implies a transitory component that is more important in the case of the MBN approach. Indeed, we verified that adding variables did not change the shape of the spectra of the MBN gaps but induced higher peaks at frequency zero. Adding more variables does not necessarily imply a more important transitory component in the case of the LRRO approach.

LRRO approach, Cochrane's approach, and the multivariate Beveridge-Nelson methodology are different in the time and frequency domains. We note, in particular, that only the output gap associated with the LRRO approach has a peak at business-cycle frequencies as defined by Burns and Mitchell (1946), that is, cycles lasting between 6 and 32 quarters. However, the estimates are imprecise for specific points in time, and it appears difficult to distinguish between these methodologies in that context. This later result is consistent with the conclusions of Staiger, Stock and Watson (1996).

Received: April 1997
Final version: July 1998

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Appendix

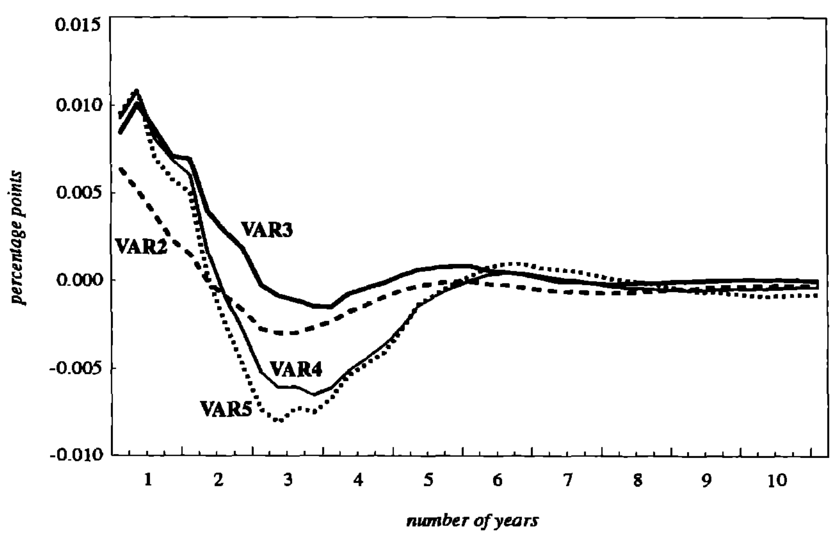

Figure A1.
Response of Real GDP to the Transitory Shocks