Online Appendix to:
"Understanding the Effect of Technology Shocks in SVARs with Long-Run Restrictions"

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We perform simulation experiments from an estimated DSGE model with US data that illustrate and support our theoretical results.[1]

1 A DSGE Model with Real Frictions

The model used for the simulations extends to one introduced in Section 2.2 by allowing for habits in consumption and investment adjustment costs. We do not claim here that the resulting estimated model is necessarily the correct data generating process for the labor productivity and hours worked, but both mechanisms have proven useful in accounting for the dynamics of macroeconomic variables in particular in terms of their persistence properties (see e.g. Beaudry and Guay, 1996, Boldrin, Christiano and Fisher, 2001 and Christiano, Eichenbaum and Evans, 2005).[2]

Intertemporal consumption choices are not time separable and the flows of consumption services are a linear function of current and lagged consumption decisions. The intertemporal expected utility function of the representative household is now given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ \log(C_{t+i} - bC_{t+i-1}) - \frac{H_{t+i}^{1+\nu}}{1+\nu} \right\}$$

where ν is the inverse of the *Frisch* labor supply elasticity. Concerning the technology side, we adopt exactly the same specification as the one adopted in Section 2.2. Remind that TFP is assumed to follow a random walk process with drift. The homogenous produced good Y_t can be used for consumption C_t and investment I_t purposes. Capital

[1] See Erceg, Guerrieri and Gust (2005), Chari, Kehoe and Mc Grattan (2008), Christiano, Eichenbaum and Vigfusson (2006) and Fève and Guay (2009, 2010) for other simulation experiments.

[2] Other specifications that allow to capture the persistence in hours have been considered in the literature: adjustment costs on labor input, as in Chang, Doh and Schorfheide (2007), Ambler, Guay and Phaneuf, (2012), learning–by–doing, as in Chang, Gomes and Schorfheide (2002), or habits il leisure choices, as in Bouakez and Kano (2006), Dupaigne, Fève and Matheron (2007), Wen (1998).

accumulation is governed by the following law of motion

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] v_t I_t$$

where $\delta \in (0,1)$ is the constant depreciation rate and $S(.)$ reflects the presence of adjustment costs. We assume that $S(.)$ satisfies (i) $S(\gamma_z) = S'(\gamma_z) = 0$ and (ii) $\xi = S''(\gamma_z)\gamma_z^2 > 0$. It follows that the steady state of the model does not depend on the parameter ξ while its dynamic properties do. As in Smets and Wouters (2007), the variable v_t represents a disturbance to the investment–specific technology process and is assumed to follow a first order autoregressive process

$$\log(v_t) = \rho_v \log(v_{t-1}) + \sigma_b \eta_{v,t}$$

where $|\rho_v| < 1$, $\sigma_v > 0$ and $\eta_{v,t}$ is iid with zero mean and unit variance.

As usual, the model is deflated for the stochastic trend component Z_t and log-linearized around the deterministic steady state to obtain a state-space representation. Let Ψ denotes the whole set of model parameters. The parameters of the state-space solution of the model depends on complicated nonlinear functions of Ψ. We split Ψ in two vectors Ψ_1 and Ψ_2. The first vector $\Psi_1 = \{\beta, \theta, \delta, \nu\}$ includes parameters which are calibrated for the US economy prior to estimation. The discount factor β is chosen such that the steady–state annual return to capital equals 3.6%. The elasticity of output to the labor input $1-\theta$ equals 0.67, which corresponds to the average share of labor income to output. The depreciation rate of physical capital δ is set equal to 0.0153. The value of $\nu = 2$ in the utility function is set according to previous estimates with US data (see Smets and Wouters, 2007).

Table 1: Parameter values Ψ

Calibrated Ψ_1		Estimated Ψ_2		
Parameter	Value	Parameter	Value	s.e.
β	0.9950	b	0.4063	0.0380
θ	0.3300	ζ	23.8476	2.6220
δ	0.0153	γ_z	1.0035	0.0008
ν	2.0000	σ_z	0.0128	0.0006
		ρ_v	0.3131	0.0659
		σ_v	0.6669	0.0743
		ρ_c	0.6893	0.1138
		c	7.4120	2.0615
		σ_c	0.0071	0.0004

Note: US quarterly data covering the sample period 1948:1–2007:4. The vector of observed data includes the growth rate of the real GDP, real consumption expenditures (non–durable & services) and hours worked (per capita).

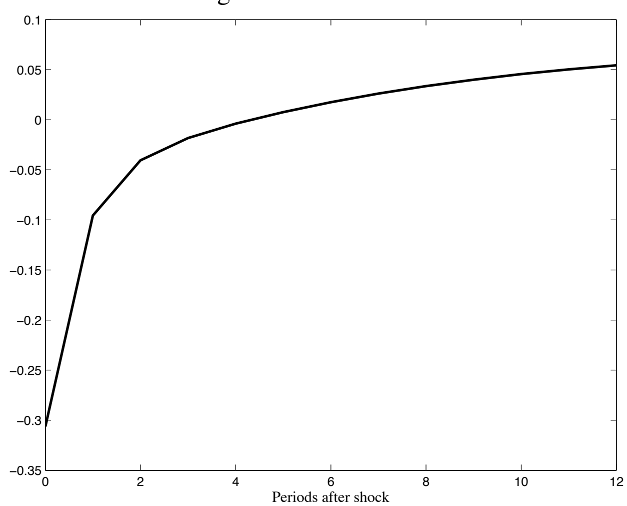
All theses values are reported in the first column of Table 1. The second vector $\Psi_2 = \{b, \xi, \gamma_z, \sigma_z, \rho_v, \sigma_v, \rho_c, c, \sigma_c\}$ contains the parameters which summarize the real frictions of the model (habits in consumption b and the dynamic adjustment cost ξ), the law of motion of the two structural shocks (γ_z, σ_z, ρ_v and σ_v) and the measurement error. As in our illustrative model, we assume that actual hours differ from those of the model by a measurement error h_t^c that follow the process

$$(1-\rho_c L)\Delta h_t^c = \left(1 - \left(1 - \frac{c}{\sqrt{T}}\right) L\right) \sigma_c \eta_{ct}$$

where $|\rho_c| < 1$, $\sigma_c > 0$ and η_{ct} is iid with zero mean and unit variance. When $c > 0$, this measurement error is non stationary in small sample (when T is fixed), but asymptotically stationary (when $T \to \infty$).

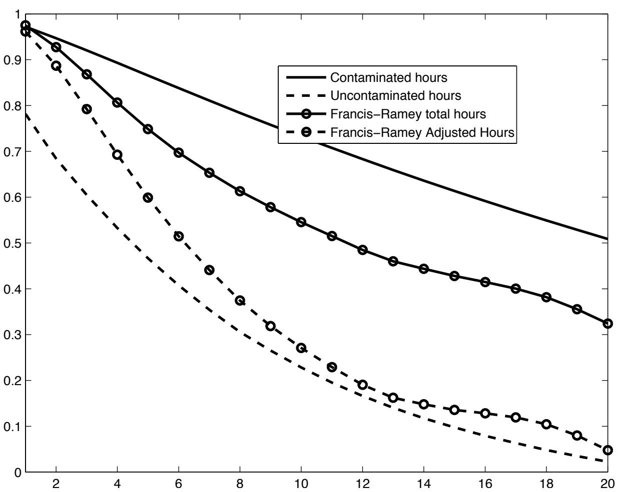
From the state–space representation resulting from the log-linearized version of the model and under the assumption of Gaussian shocks, the log–likelihood function can be evaluated. The parameters of vector Ψ_2 are then estimated by maximizing this function. We use US quarterly data covering the sample period 1948Q1–2007Q4. The observed variables are the growth rate of the real GDP, real consumption expenditures (non–durable & services) and total hours worked (per capita). Total hours worked are borrowed from Francis and Ramey (2009). The estimation results are reported in the second column of Table 1. The parameters are precisely estimated and are in the line of previous estimations for the US economy (see Smets and Wouters, 2007). The habit persistence parameter b is positive and significant. The adjustment cost parameter ξ takes a large value. These estimated values are crucial in replicating US data, especially the serial correlation of output growth and the log of hours. For example, setting $b = \varphi = 0$ dramatically reduces the log–likelihood and a likelihood ratio test strongly rejects this restriction. In other words, our estimation results favor a version of the model with a sizeable amount of real frictions. The investment shock exhibits small persistence but its standard error is significantly higher than the one for the permanent technology shock. Finally, the estimated parameters of the measurement error rejects a pure random–walk representation.

Figure 1: IRF of Hours



Using estimated values, we compute the dynamic responses of hours worked following a technology shock implied by the model (see Figure 1). Hours worked decrease on impact and its response turns out to be very persistently positive after one year. This means that the model by itself is able to generate persistent fluctuations in hours. These findings are again in accordance with those obtained from estimated DSGE models (see Smets and Wouters, 2007), from SVAR models (see Galí, 1999, Francis and Ramey, 2005) and direct measures of TFP (see Basu, Fernald and Kimball, 2006). In our model, this response of hours is the result of the interplay between habit persistence in consumption and adjustment costs on investment. As pointed out by Francis and Ramey (2005), strong enough habit persistence induces a sluggish response of consumption. Facing a positive technology shock, households can put the extra resources on investment. However, the high degree of adjustment cost on capital implies that an additional investment is very costly. Consequently, households choose to spend their new wealth on the only remaining choice, *i.e.* they increase their leisure. We also use the estimated DSGE model in order to compute some statistics which summarize the time series behavior of two measures of hours worked: the first, labeled *contaminated hours*, includes the measurement errors and the second, labeled *uncontaminated hours*, is directly obtained from the estimated DSGE model. Consequently, we obtain two measures of labor productivity depending on the hours worked measure. We evaluate the contribution of the technology shock to labor productivity growth and change in hours worked. This shock accounts for a small portion of fluctuations in contaminated hours worked since it represents 12.43% of their variance. Interestingly, the technology shock explains 51.19% of the labor productivity growth. This is in contrast with the uncontaminated measure of hours. In this case, the technology shocks accounts for 73.04% of labor productivity growth.

Figure 2: ACFs



The computation of the autocorrelation function of both measures of hours are reported in Figure 2. For comparison purpose this figure includes the autocorrelation function of total and adjusted hours of Francis and Ramey (2009). This figure clearly shows that the contaminated measure of hours displays more persistence than the uncontaminated measure, in accordance with the actual data reported in Francis and Ramey (2009).

2 Simulation Experiments

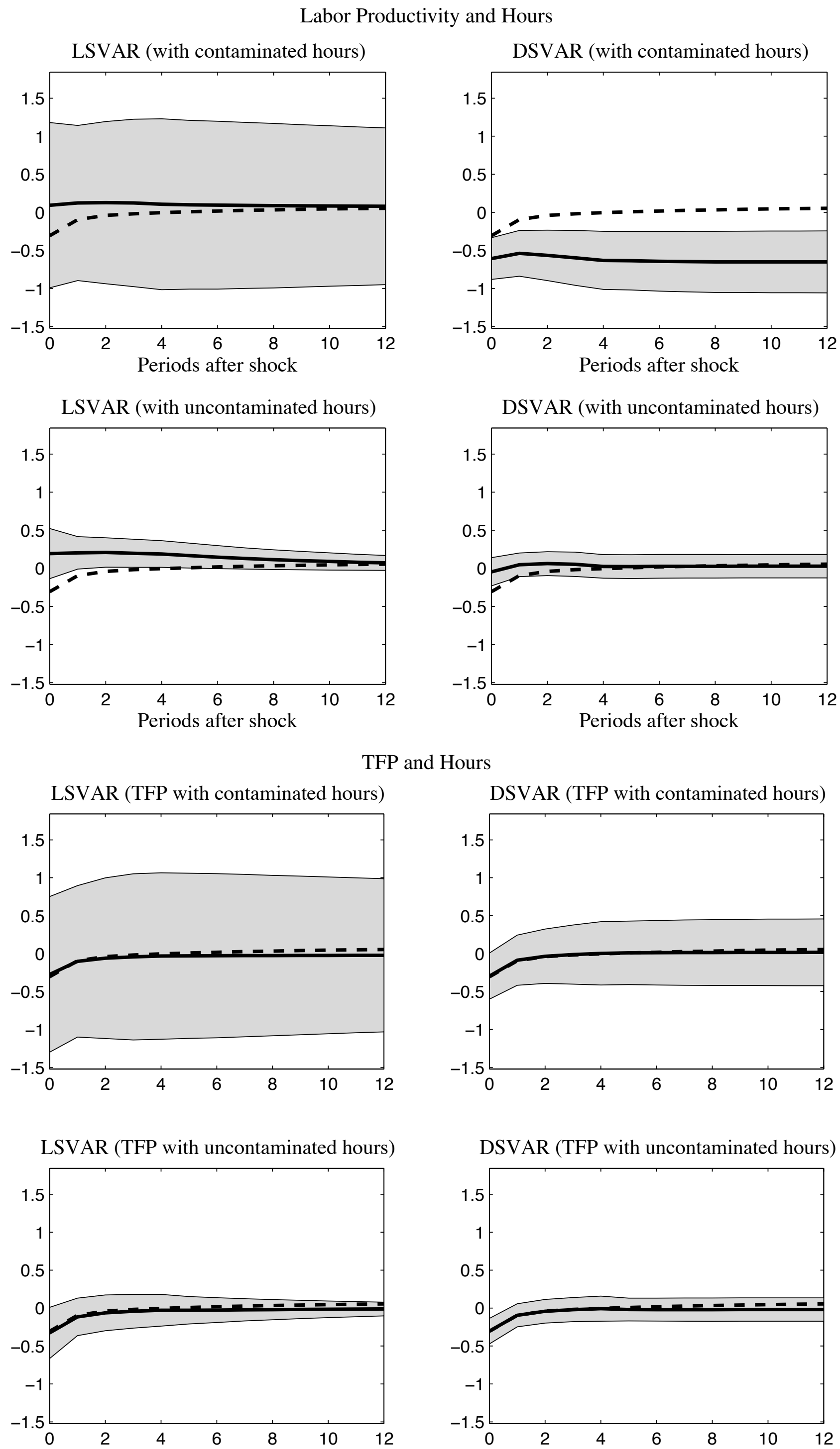
We now use the model to simulate artificial data, over which we replicate the different structural VARs used in the relevant literature and in the empirical part of the paper. To compute artificial time–series of the variables of interest, we draw $S = 1000$ independent random realizations of the TFP innovation η_{zt}, the investment–specific technology innovation η_{vt} and the measurement error innovation η_{ct}. Using the parameters of Table 1, we compute $S = 1000$ equilibrium paths for the growth rates of labor productivity and hours worked. In all experiments, the sample size is equal to 240 quarters, as in actual data. In order to reduce the influence of initial conditions, the simulated sample includes 250 initial points which are subsequently discarded before the estimation of VAR models. For each draw, the number of lags in VAR models is set to 4, a value typically used in empirical studies. The results are reported in Figures 3–6.

Let us first consider the SVAR models that include labor productivity and hours (see the top panel of Figures 3 and 4). Our main results are the following. First, the response of hours obtained from the DSVAR model displays a downward bias when the measure of hours is contaminated and this bias lasts in long-run (not reported in this Figure to save space). This bias decreases when the low frequency movements are removed from hours. Second, the measurement of hours (contaminated and uncontaminated) in the LSVAR model does not affect the estimated response (the LSVAR over–estimates the true response). Third, the confidence interval is smaller and decreases as the horizon of the response increases when uncontaminated hours are used instead the contaminated ones. Obviously, uncontaminated hours are less volatile than the contaminated ones, since the latter includes measurement errors. However, we redo the same exercise adjusting for the standard errors of the technology and shock investment shock, such that the volatility of uncontaminated hours equals the one of contaminated hours. We then include this new series in the LSVAR model and compare the response to the LSVAR with contaminated hours, as reported in the top panel of Figure 3. We obtain a large confidence interval for the estimated response, but a decrease with the horizon of the response. This is in contrast with the case of contaminated hours; for which the confidence interval does not decrease with the horizon. We have also investigated the case of stationary measurement errors (we set $c = 0$). We adjust the volatility of the measurement error shock (to get similar variance of hours) and then compute contaminated hours. Again, we obtain a confidence interval that decreases when the horizon decreases. All these findings echoes our analytical results.

We now consider the SVAR models that include TFP and hours (see the bottom panel of Figures 3 and 4). Now, the first variable which is used to identify a technology shock is not polluted by the low–frequency movement in the contaminated hours. In this case (as shown in the Proposition 1), the low–frequency movements in contaminated hours only affect the long–run variance of hours worked. The figures show that when the econometrician uses a proper measure of the technology, the specification of hours in the VAR model does not matter. Each SVAR consistently estimates the dynamic effects of technology shocks on hours. At the same time, the previous results apply: we obtain a smaller confidence interval if uncontaminated hours are included in the VAR model and this confidence interval decreases with the horizon of the response.

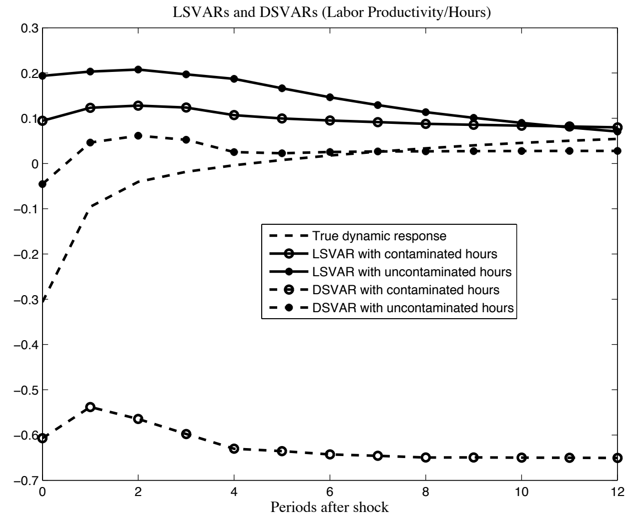
Finally, we investigate the dynamic responses of the productivity measures (labor productivity, TFP) to a technology innovation. These are reported in Figures 5 and 6. Following Proposition 1, the DSVAR model using labor productivity with a contaminated hours must lead to over–estimate the true long-run response. This is reported in the bottom of the Figure 5. As previously mentioned, two shocks increases permanently the labor productivity (technology shock and measurement error) and this corrupts the long–run identification strategy. Notice that the estimated short-run responses differs significantly from the true ones and the long-run impact is significantly over-estimated as expected (not reported in the Figure but available on request). When these low–frequency movements are removed from hours worked, the DSVAR model delivers dynamics responses close to the true one. The true responses lies now in the confidence interval of the estimated response. The simulation experiments with the LSVAR model does not

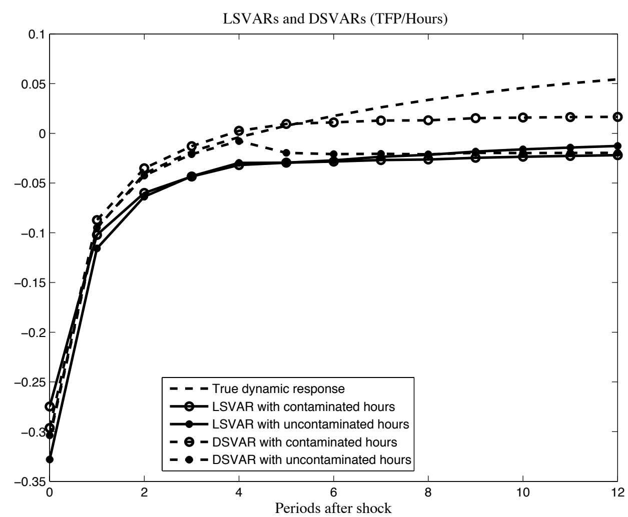
Figure 3: IRF of Hours: Measures of Productivity and Hours



Note: The DSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated) in first difference. The LSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated). The selected horizon for IRFs is 13. The number of lags in VAR model is 4. The dashed line corresponds to the true IRF of hours. The solid line corresponds to the estimated IRFs from SVARs. 95 % confidence interval shown.

Figure 4: IRF of Hours: Comparison of SVARs

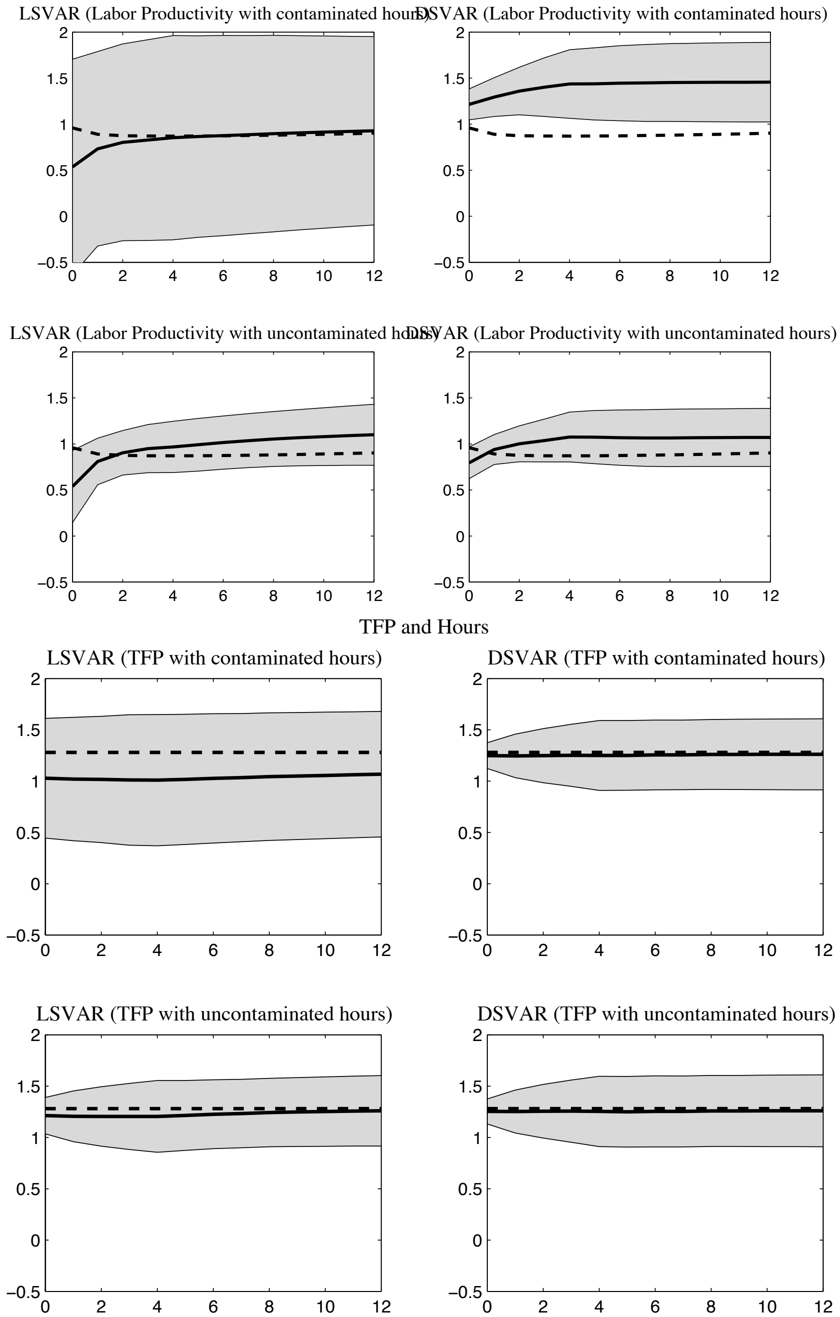




Note: The DSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated) in first difference. The LSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated). The selected horizon for IRFs is 13. The number of lags in VAR model is 4.

Figure 5: IRF of Productivity

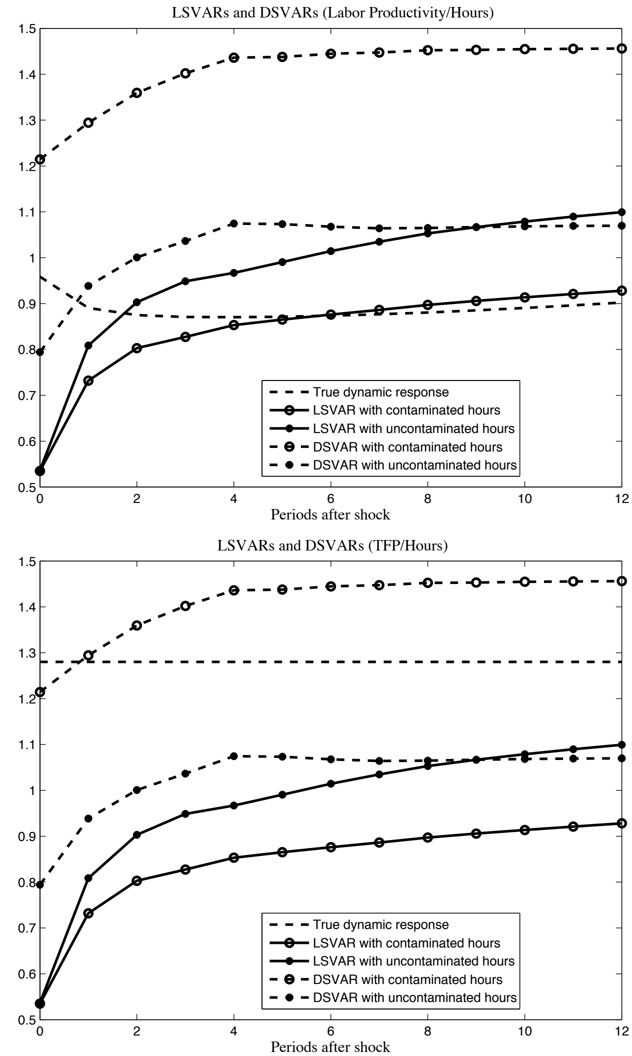
Labor Productivity and Hours



Note: The DSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated) in first difference. The LSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated). The selected horizon for IRFs is 13. The number of lags in VAR model is 4. The dashed line corresponds to the true IRF of hours. The solid line corresponds to the estimated IRFs from SVARs. Confidence intervals are based the 95-percentile from 1,000 Monte-Carlo experiments.
95 % confidence interval shown.

deliver clear cut evidence. It appears that the estimated response is less biased than those obtained from the DSVAR model and the results seems less sensitive to low frequency movements in hours worked. When the productivity measure in the VAR model is now the TFP, SVAR models yield more accurate dynamic responses.

Figure 6: IRF of Productivity Measure: Comparison of SVARs



Note: The DSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated) in first difference. The LSVAR model includes labor productivity growth or TFP growth and the log of hours (contaminated and uncontaminated). The selected horizon for IRFs is 13. The number of lags in VAR model is 4.

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